Models for Wellbore Leakage

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Outline

• Challenges of the Well Leakage Problem
• Our Modeling Approach
• Numerical, Analytical, and Semi-analytical Models
• Concluding Comments
Worldwide Density of Oil and Gas Wells

Number of Wells Drilled per ~10,000 km²

1 - 100  100 - 300  300 - 1,000  1,000 - 4,400  4,400 - 23,400  23,400 - 61,000  No Wells/Data

End of 2004

From IPCC SRCCS, 2005
Injection and Leakage

• How to model this system?
• Domain Size: 1,000 km²
• Leakage Pathways: 0.001 m².
• Flow Properties along well highly uncertain.
• Possible Material Degradation.

(From Duguid, 2006)
Our Approach to Modeling

• Simplify the system (but not too simple)
  – Macroscopic sharp interface (buoyant segregation)
  – Vertical equilibrium / Structured vertical velocity
  – Focus on early time $\rightarrow$ Max risk of leakage
    • Two-phase flow physics dominates
    • Ignore geochemistry, non-isothermal effects

• Develop very fast analytical, semi-analytical, and hybrid numerical-analytical solutions.

• Apply simulation tools in a Monte Carlo framework.

• Combine models into 'hierarchical' framework

Numerical Solutions

Solve for $p(x,y,t)$, $h(x,y,t)$
Analytical Solution

\[
\frac{dh'}{d\chi} = 4\Gamma \frac{\gamma_1}{\chi} \frac{d}{d\chi} \left( (1-h') \chi \frac{dp'}{d\chi} \right)
\]

\[- \frac{di'}{d\chi} = 4 \gamma_2 \Gamma \lambda_1 \frac{d}{d\chi} \left( i' \chi \frac{d}{d\chi} (p' + h' + \vartheta i') \right) \]

\[- \frac{d}{d\chi} (h'-i') = 4\Gamma \frac{\lambda_2}{\chi} \frac{d}{d\chi} \left( (h'-i') \chi \frac{d}{d\chi} (p' + h') \right) + 4(1-\gamma_2) \Gamma \lambda_1 \frac{d}{d\chi} \left( i' \chi \frac{d}{d\chi} (p' + h' + \vartheta i') \right) + 4\Gamma(1-\gamma_1) \frac{d}{d\chi} \left( (1-h') \chi \frac{dp'}{d\chi} \right) \]

\[\chi \equiv \frac{r^2}{\tau} \]

(From Nordbotten and Celia, *JFM*, 2006; See Celia and Nordbotten, 2009)
Similarity Solution: Simplified

When $\Gamma < 0.5$

$$h'(x) = \frac{h(x)}{H} = \frac{1}{\lambda - 1} \left( \sqrt{\frac{2 \lambda}{x}} - 1 \right)$$

$\chi_{\text{max}} = 2 \lambda$

$\chi_{\text{min}} = 2 \frac{1}{\lambda}$

(From Nordbotten and Celia, *JFM*, 2006)
A Semi-analytical Model

1. Injection Plume, Secondary Plumes and Pressure Fields: Similarity Solution *(Nordbotten and Celia, JFM, 2006)*

2. Leakage Dynamics: Multi-phase Darcy Flow along Leaky Well Segments *(Nordbotten et al., ES&T, 2005, 2008)*

3. Upconing around Leaky Wells *(Nordbotten and Celia, WRR, 2006)*

4. Grid-free solutions: We can now solve 50 years of injection over 2,500 km², 12 layers, and 1,200 wells in about 15 minutes.

\[ Q_{\text{well}} \propto K_{\text{well}} k(S_\alpha) \left( \frac{P_1 - P_2}{H} - \rho_\alpha g \right) \]
Study Area around Edmonton – Wabamun Lake
Model Results

Basal sandstone

Nordegg

Nisku
Recent Developments

• High-performance Implementation (Elsa)
  – Complete re-implementation of code in C++
  – Highly modular, very efficient

• Expanded Physics in Semi-analytical Model
  – Diffuse leakage of brine through caprock formations
  – Improved similarity solutions for low flow rates

• User-friendly Interfaces
  – Web-based interface for simple systems
  – Multiple formats for input

• Separate numerical sharp-interface code (VESA)

• Designs for a hierarchical modeling platform.
Concluding Remarks

- Simplified models can be reasonable because:
  - Buoyancy provides strong vertical segregation
  - Space- and time-scale separation for critical processes
  - Large uncertainties in critical leakage parameters make detailed fine-scale simulation unnecessary

- Fully coupled detailed models are appropriate for:
  - Fine resolution along critical leakage pathways
  - Computational upscaling for bulk parameters
  - Basic Science investigations

- Important practical questions require practical models.
Thank You!
Publications


Critical Parameters

- Reservoir Formations (Upscaled):
  - Permeability ($k$), Porosity ($\phi$), and Thickness ($H$)
  - Residual Saturations ($S_{res}$)
  - Endpoint Relative Permeability ($k_{rel}$)

- Caprock Formations:
  - Permeability
  - Thickness
  - Preferential Flow Paths

- Old Wells (and Faults):
  - Depth
  - Effective Permeability ($k_{well}$)
  - Geochemical reactions, other local nonlinear processes